Modelling of Firm Offer from Combined Wind and Hydro Generations

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***Abstract*—This paper analyses the impact of a firm combined offer by wind and small hydro generators located in the river chain, with a view to address the intermittency of wind generators. Both generations are dispatchable and cleared against their offer prices. They offer a firm, hourly-schedule (WH schedule) for 24 hours ahead of real-time operation to an auction based locational marginal price (LMP) market with other generators offering to meet the system loads. The model network consists of other generators and loads at different buses. The scheduled power is taken off at a predetermined bus, as a load at the bus. This schedule must be met by the wind and hydro combined generations. If necessary, a notional thermal generation is available at a considerable higher price to meet the schedule, at the offtake point.**

The objective is to minimise the total supply cost for 24 hours and examine LMPs and constraint-on costs while respecting the given WH schedule, nodal power balance constraint, generation limits, branch flow and other limits. Discussion is based on New Zealand (NZ) Electricity Market rules, where generators are self-committed into the market. Three scenarios are studied and the results presented.

***Index Terms*—Constraint-on payment, Dual Problem, Electricity market, Firm offer, Hydro generation, Optimisation.**

I. NOMENCLATURE

$c_{gi,t}$	Energy offer price by generator *i*, \$/MWh, at time t.
$P_{gi,t}$	Cleared MW generation at bus *i*, at time t.
$unmet_{i,t}$	Infeasible generation, MW at bus *i* at time t.
$P_{di,t}$	Demand at bus *i*, MW at time t.
$P_{i,j,t}$	Power flow in branch between *i* and j, MW at time t.
$B_{i,j}$	Branch admittance between *i* and j, in pu
$a_{i,t}$	Bus angle in radian at bus *i* at time t.

Greek letter appearing on the right hand side of each constraint is/are the dual variable(s) or shadow price(s) associated with the constraints.

II. INTRODUCTION

Wind generation is a first choice as a source of clean and green energy. However, wind's intermittent nature limits its utility for unit commitment purposes in generation scheduling. Most wholesale markets treat wind generators as "must run" (MR) units for unit commitment. Unable to provide firmed capacity wind generation is generally unable to establish the system clearing price and the associated higher levels of remuneration.

The trend in many jurisdictions for wind generation to form an increasing proportion of capacity will accelerate the need to make wind generators "dispatchable" units.

NZ has set a target of 90% of renewable energy by 2025 and it is expected that the installed wind generation capacity of 700 MW will increase at least 1000 MW more by 2025.

A number of proposals to provide firmed output by combining wind generation with storage have been discussed. The authors of [1] discussed pumped storage as a mechanism to create the storage. The authors of [2] addressed environmental constraints and unit commitment problems. A paper [3] discussed the value of combining energy storage and wind in short term energy and balancing market. The author of [4] discussed wind and hydrogen storage. The authors of [5] have addressed small storages associated with a hydro generator river chain to firm wind generation within the environmental and resource constraints of such a chain. This paper examines the effect of the WH constraint that says that the combined wind and hydro generation must meet their own pre-submitted schedule at a certain bus, and the associated constraint-on payments using dual model. Next it shows the effect of nodal price on the transmission congestion. Then it finally examines the difference in costs when wind generators are treated as must run, with free entry to the market with zero offer price.

Procedure and data are described in section 3. Section 4 presents an analysis based on mathematical optimization primal and dual models. Three scenarios are studied and their results are discussed in section 5, conclusions and references follow.

III. Procedure and Data

A. *Procedure*

First, a wind-hydro (WH) schedule is created for different hours which wind and hydro generators agree to deliver to the market. Wind and hydro generators offer into the market for 24 hours (MW, $/MWh). A notional thermal generator supplies the shortfall between the WH load schedule and combined wind and hydro generations, in each dispatch interval at a considerable higher prices. The cost of supplying the scheduled load using wind, hydro and thermal generators, and the total system load for each hour, is minimized. Transmission network is introduced as a bus-branch model. There are other generators, loads at different nodes in the network in addition to the specified WH schedule, to be delivered at load bus 8. The objective minimizes the hourly interval cost and total cost of dispatch over 24 hours, subject to a number of constraints in each time period, e.g. generation limits, transmission branch limits, nodal power balance constraints, and WH schedule constraints met by wind, hydro and thermal generations.

B. *Load and GenerationOffer Price Profiles*

Fig. 1 shows the energy offer prices for wind and hydro energy for each period. It shows the hourly energy offer prices in $NZ/MWh. The hydro energy price is a representative two days' average at the South Island HVDC bus where there is a large hydro station. A representative wind energy price for dispatchable generation is also shown in Fig. 1. The wind energy price is based on assumptions and relative to hydro generation price. Thermal energy offer price is $100/MWh for all hours. The hourly bus loads at different buses are shown in table 1. Total system load is shown in the last column of the table.

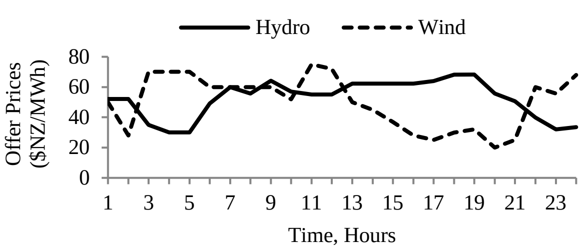

Fig. 1. Hourly Hydro and Wind energy offer prices

1) *WH schedule and generation offers*

Column "8" of table 1 shows the wind-hydro (WH) load schedule for the combined wind and hydro generators to meet at load bus 8. This schedule is designed to test the optimization through a range of wind, hydro and deficit generator outputs. Fig. 2 shows all generators' offer prices including hydro and wind generators that are shown in Fig. 1. The generator buses are 1, 3(hydro), 7 (thermal), 11, and 13 (wind), see Fig 3.

2) *Network*

A lossless 16-bus network is considered as shown in Fig.3. It consists two areas connected by a branch between bus 4 and bus 8. Each branch has an admittance of 50pu and a capacity of 1000 MW in both directions. Branch 4-8 has a capacity of 1500 MW, reduced to 1300 MW to make the branch constraint bind.

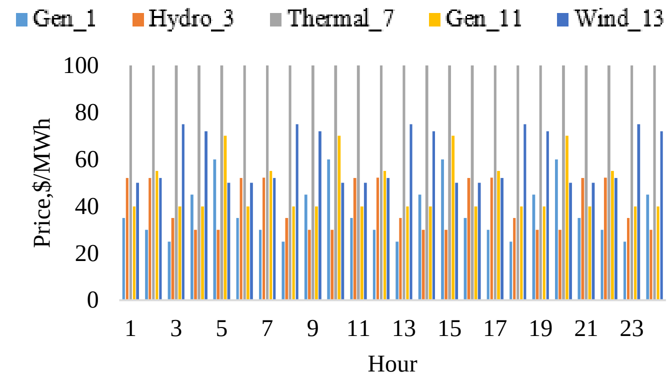

Fig. 2. Generator offer prices ($/MWh)

TABLE I. BUS LOADS (MW) FOR 24 HOURS

Hour	2	5	6	8	9	10	12	14	15	16	Total
1	100	200	150	350	200	100	200	200	20	20	1540
2	120	220	200	400	220	150	220	220	30	30	1810
3	140	240	200	480	240	170	240	250	20	20	2000
4	150	250	220	540	260	180	250	270	25	25	2170
5	170	280	230	650	280	200	270	280	10	10	2380
6	200	300	250	750	300	220	300	300	20	20	2660
7	220	300	260	800	320	230	310	320	30	30	2820
8	250	310	270	860	330	240	320	320	20	20	2940
9	260	320	280	900	340	250	330	330	25	25	3060
10	280	330	300	900	350	260	350	350	10	10	3140
11	300	310	300	875	360	270	350	350	20	20	3155
12	300	320	300	850	330	250	300	300	30	30	3010
13	280	300	280	800	300	230	280	270	20	20	2780
14	270	280	250	740	300	210	270	260	25	25	2630
15	260	270	240	720	280	200	260	250	10	10	2500
16	270	300	250	725	300	220	280	270	20	20	2655
17	280	310	260	730	310	230	290	280	30	30	2750
18	300	320	270	800	320	240	300	290	20	20	2880
19	320	330	280	860	330	250	320	300	25	25	3040
20	350	340	290	840	340	260	350	320	10	10	3110
21	300	300	270	800	300	240	320	300	20	20	2870
22	280	280	250	670	280	230	300	300	30	30	2650
23	220	250	200	450	270	220	280	270	20	20	2200
24	150	210	160	380	280	220	250	250	25	25	1950

3) *Generation maximum and minimum MW limits*

Table 2 shows the generator numbers in the 1st row, and their minimum and maximum MW limits, in 2nd and 3rd rows respectively.

TABLE II. Generator Limits (MW)

GEN	1	3	7	11	13
P min	0	0	0	0	0
P max	1500	600	60	1500	270

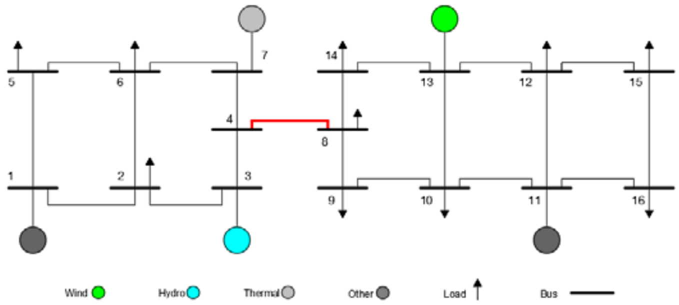

Fig. 3. 16 Bus Network

IV. OPTIMIZATION PROBLEM

A. *Primal Mathematical model*

The model's objective is to minimise the total generation costs over 24 hourly intervals (1).

$$\text{Minimize: } Z = \sum_{t=1}^{24} (C_t) \tag{1}$$

Minimise cost for each interval (2).

$$C_t = \sum_{i=1}^{n} \left(C_{gi}\, P_{gi}\right) \ \forall i \tag{2}$$

The load at bus 8 (WH schedule) must be met by wind, hydro and if necessary by thermal generations (3).

$$P_D(\text{"8"}) = \sum_k P_{gk}\ ;\ \eta_8 \tag{3}$$

The constraints (4, 5) express the forward and reverse flow of each branch.

$$P_{i,j} = B_{i,j} \,.\left(a_i - a_j\right);\tau_{i,j}, j > i, \forall (i,j),\ \ j \in N_i \tag{4}$$

$$P_{j,i} = -P_{i,j}\ ;\psi_{j,i}\,, \qquad j > i, \forall (i,j), j \in N_i \tag{5}$$

Power balance at each bus must be respected (6).

$$P_{gi} - P_{di} = \sum_j P_{i,j};\ \lambda_i, \qquad j \in N_i, \forall i \tag{6}$$

Demand is set at each bus (7).

$$P_{di} = P_{di}^{set};\, \beta_{i.}\ \ \forall i \tag{7}$$

Slack bus angle is set at zero (8).

$$a_i = 0\ ;\ \pi_i, \qquad i = Slack\ bus \tag{8}$$

The constraints (9- 12) express the transmission branch flow and generation limits.

$$-P_{ij} \geq -P_{ij}{}^{max};\ \phi_{ij}{}^{+}\ ;\ j > i \tag{9}$$

$$P_{ij} \geq P_{ij}{}^{min}\ ;\ \phi_{ij}{}^{-}\ ;\ j > i \tag{10}$$

$$-P_{gi} \geq P_{gi}{}^{max};\ \mu_i{}^{+} \tag{11}$$

$$P_{gi} \geq 0\ ;\ \mu_i{}^{-} \tag{12}$$

Positive variables: P_{gi} , P_{di}

$j \in N_i$ Represents all nodes connected to bus i, j: {there is a line between i and j}. Unmet generation in (6) is not shown.

B. *The Lagrangian*

$$\begin{aligned}
L = &\sum_{i\in N} c_{gi}\, P_{gi} + \sum \lambda_i \Big(\sum_{j\epsilon Ni} P_{ij} - P_{gi} + P_{di}\Big) \\
&+ \sum_i \beta_i \,(P_{di}^{set} - P_{di}) + \sum_k \eta_8 \Big(\sum_k P_{gk} - P_D(\text{"8"})\Big) \\
&+ \sum_{ij;j>i,j\in N_i} \tau_{ij}\big[B_{ij}(a_i - a_j) - P_{ij}\big] + \sum_{ij;j>i,j\in N_i} \psi_{ji}(-P_{ij} - P_{ji}) \\
&- \sum_i \pi_i(a_i) + \sum_{ij;j>i,j\in N_i} \phi_{ij}^{+}\,(P_{ij} - P_{ij}^{max}) - \sum_{ij;j>i,j\in N_i} \phi_{ij}^{-}\,(P_{ij}^{min} - P_{ij})
\end{aligned} \tag{13}$$

C. *The dual constraints*

The dual constraints are formed from the first order optimality condition known as the Karush-Kuhn-Tucker (KKT) condition. On the right hand side of the dual constraint are the primal variables. Differentiating L with respect to each primal variable (indicated to the right of the corresponding dual equation), we get,

$$c_{gi} - \lambda_i + \gamma_i = 0\,; P_{gi};\ \gamma_i = \gamma_i^{+} - \gamma_i^{-} \tag{14}$$

$$\lambda_i - \beta_i - \delta i * \eta_8 = 0; P_{di}; \delta i = 1\ if\ i = 8; \text{else } \delta i{=}0 \tag{15}$$

$$-\tau_{ij} + \lambda_i + \phi_{ij} - \psi_{ji} = 0 \quad : P_{ij}; j > i \tag{16}$$

$$\phi_{ij} = \phi_{ij}^{+} - \phi_{ij}^{-} \tag{17}$$

$$\lambda_j - \psi_{ji} = 0 \quad : P_{ji} \tag{18}$$

$$\sum_{j>i} \tau_{ij} B_{ij} - \sum_{j<i} \tau_{ji} B_{ji} = 0 \quad : a_i; i \neq 1 \tag{19}$$

$$\pi_1 = 0 \quad : a_1;\ \text{s} = 1 \tag{20}$$

$$\gamma_{bi} = \gamma_{bi}^{+} - \gamma_{bi}^{-}; \quad \phi_{ij} = \phi_{ij}^{+} - \phi_{ij}^{-} \tag{21}$$

From (19) and (20), we get

$$\lambda_j - \lambda_i = \phi_{ij} - \quad \tau_{ij} \tag{22}$$

Notice that we considered conventional flows in the forward direction $P_{ij}(j > i)$ in (4) and reverse flows in $P_{ji}(j > i)$ in (5). Their dual variables are respectively τ_{ij} and ψ_{ji} and both of them are associated with equality primal constraints and therefore unconstrained in sign. The value associated with the line capacity is defined by the dual variable where $\phi_{ij} = \phi_{ij}^{+} - \phi_{ij}^{-}$ both ϕ_{ij}^{+} and ϕ_{ij}^{-} are non-negative. If the flow is not constrained either at the upper bound or at the lower bound, then $\phi_{ij} = 0$. It indicates that there is no need to increase the capacity and there is no value in increasing the capacity. If the flow is binding at its upper bound, then $\phi_{ij}^{+} > 0$. This shows that given one more unit of line capacity in that direction we should be able to reduce the system operating cost (objective function). Notice that we expressed the right hand side of the primal constraint (9) as minus the line capacity, and since ϕ_{ij}^{+} equals the change in the cost of running the system resulting from an increase in right hand side resources (capacity), ϕ_{ij}^{+} actually measures the marginal value of decreased line capacity. Hence a decrease in line capacity increases the system operating cost. So increased line capacity must decrease the cost. Therefore ϕ_{ij}^{+} indicates the value of having one more unit of line capacity. Similarly, if flow is binding at its lower bound in that direction then - $\phi_{ij}^{-} < 0$. This indicates that if the lower bound were raised by one unit in that direction then the cost of operating the system would increase. Similar interpretations apply for the bounds on generation capacity constraints.

V. STUDY RESULTS

The study was conducted for the following scenarios using a program written in GAMS [6] and the “EXPRESS” LP solver.

Three cases are examined using a 16-bus network. In these cases hydro, thermal and wind generation will be injected at buses 3, 7 and 13; and withdrawn at bus 8. These generators meet the committed WH load schedule and take part in the optimization with all other generators and meet the network constraints and bus loads. Note that the wind generation is treated as “dispatchable” and cleared against its offered energy prices. The following 3 cases are studied.

- Case 1- No transmission constraint
- Case 2 - Case 1 with transmission constraint
- Case 3 - Case 1 but wind generators are treated as “Must Run”, Free entry to the market

A. *Case 1 – No Transmission Constraint*

1) Dispatch

Dispatch is shown in Fig. 4. The generators at buses 3 and 13 meet the WH schedule for all the hours except during hours 9, 10, 11. Thermal generation of 30 MW was required during hours 9 and 10; and 5 MW during hour 11.

2) Bus Prices

The price at bus 1 during 24 hours is shown in Fig. 5. The price on other buses at each hour is same, because there is no binding transmission constraint and no loss. The following observations on price formation can be made.

In the primal problem, we considered constraint (3) which says that the load at bus 8 must be supplied by the sum of generations at buses 3, 7, and 13. The constraint (3) is an equality constraint and thus the value of its shadow price (dual variable) is unrestricted in sign. This constraint is actually a bit complex because it links demand, itself a variable, to generations of some specific generators that are also variables. So the right hand side of the constraint is actually zero. What shadow price of this constraint reporting is in fact the cost of changing the RHS of the constraint to 1 unit which increases sum of net group generation by 1 unit relative to demand at bus 8. This shadow price effectively expresses the constraint-on payment to these generators (difference between the marginal generation cost and the nodal price).

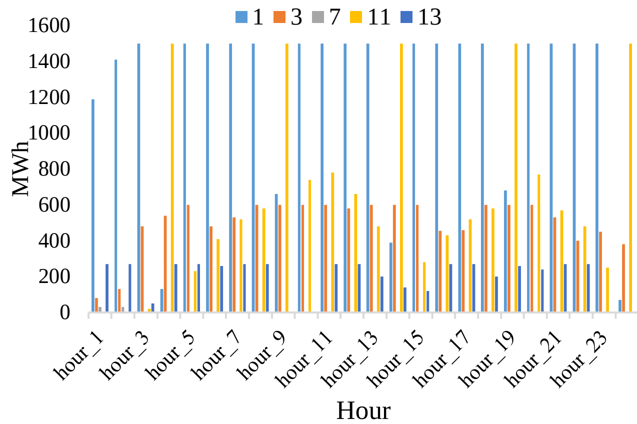

Fig. 4. Dispatch at different hours

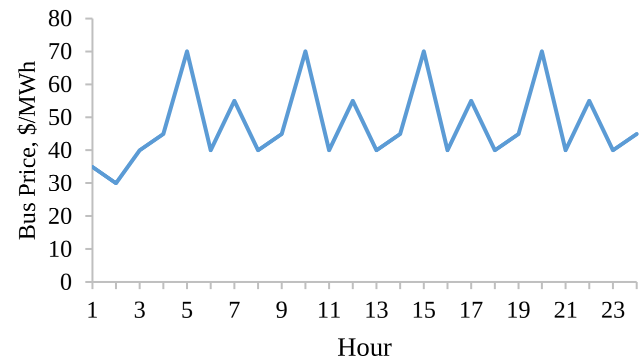

Fig. 5. Nodal price at Bus 1

3) Price Analysis

TABLE III. DECOMPOSITION OF BUS PRICES

Bus= 2, Hour = 3; η_8 = 0, β_i =40, λ_i=40			
G#	$/Mwh	Capacity	Dispatch
1	25	1500	1500
3	35	600	480
11	40	1500	20
Bus = 8 , Hour = 8; η8 = -35, βi =75, λi =40			
1	25	1500	1500
3	35	600	600
7	100	60	
11	40	1500	580
13	75	270	260
Bus = 8 , Hour = 10; η8 = -29.95, βi =100, λi=70.05			
1	25	1500	1500
3	35	600	600
7	100	60	30
11	40	1500	740
13	75	270	260

The nodal price at the load buses can also be obtained by β_i. using dual constraint (15). The shadow price β_8, at the load bus 8 captures the marginal cost of group generation for supplying the load at the bus. The nodal price at the load buses is given by dual constraint (15), i.e. $\beta_i = \lambda_i - \eta_8$; $\eta_8 = 0$ for all bus except for bus 8. Graphically, the gap between the two curves represents the constraint-on payment as shown in Fig 6. Also see table3. It is readily available from the dual variable associated with (3), η_8. It can be seen that constraint-on payment can be positive or negative.

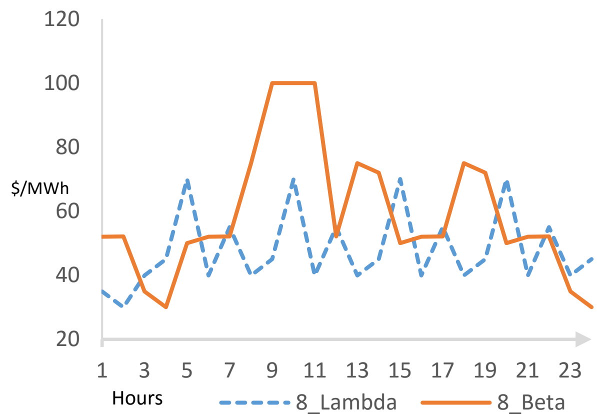

Fig. 6. Constraint-on Payment (Lambda-Beta)

B. *Case 2 - Case 1 with transmission constraint invoked*

1) Bus Prices

The rating of branch 4-8 is reduced from 1500 to 1300 MW, in this case in order to make the constraint binding. The constraint is binding during three hours. These are hours 3, 5 and 15 with constraint price of $15, $10.05 and $10.05. Bus prices at hours 3, 5 and 15 are shown in Fig. 7.

The transmission congestion is given by (Loss-less network) the difference in nodal prices across the branch as shown in equation (22).

The bus prices, during hour-3, are $25/MWh, and $40/MWh at the two sides of the binding constraint 4-8. The difference of price between bus 4 and 8 is explained by the constraint price of $15.

The prices on buses 1-7 decreased by $10.5 during hours 5 and 15 compared to $70.5 in case 1. The prices on buses 8-14 increased by $15 during hour 3 compared to $25 in case 1. The prices on all buses during all other hours remain same as in case 1 (Fig.7).

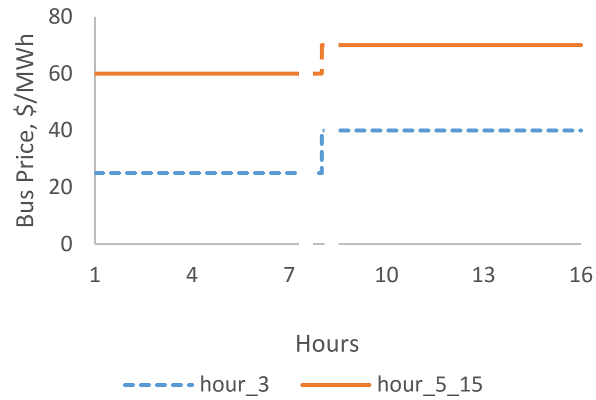

Fig. 7. Bus prices when the branch 4-8 is congested (Case 2)

2) Difference in Dispatch Case 2 and Case 1

The difference in dispatch between cases 1 & 2 is shown in Fig.8. While branch constraint 4-8 is binding, the generation at the area where bus 8 is located, at bus 11 is increased in case 2, compared to case 1, during hours 3, 5 and 15. There is no change in dispatch in generations at bus 11 during other hours, as shown in Fig. 8.

It is also evident from Fig. 8 that the generation at bus 1, is decreased during hours 3, 5 and 15 in case 2 compared to case 1, while the constraint is binding. The decreased generation has been picked up at generation at bus 11.The wind, hydro and thermal generations did not require change meeting the committed load during these hours.

The total dispatch cost for 24 hours is $2583237, which is slightly higher (<1%) than that in case 1, because of re-dispatch.

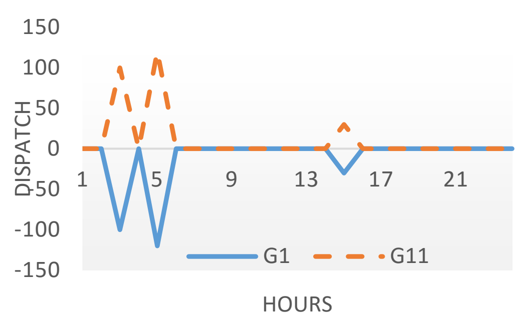

Fig. 8. Difference in dispatch

C. *Case 3 - Case 1with no offer price for wind generation*

The case 1 was re-run with zero offer price from the wind generators. The result shows that the generation cost reduced by $537105(21%) as compared to Case 1, as expected. All the offered wind generation at different hours are dispatched, as required. The bus prices are same as that in case 1.

VI. COMPARISON OF 3 CASES

The dispatch cost for 24 hours, MW dispatch, and bus prices for three studied cases, are shown in table 4.

TABLE IV. COMPARISON OF 3 CASES

Cases	Cost	Dispatch	Bus Price
Case 1	$2,580,230	Base case	Marginal bus price is same for all buses, for the hour
Case 2	$2,583,237	Generations are re- dispatched as shown in Fig. 8.	Price separation takes place, as shown in Fig. 7
Case 3	$2,043,125	All wind generations are dispatched. Thus total dispatch cost is reduced.	Marginal bus prices remain same as in case 1.

VII. CONCLUSIONS

In this paper, primal optimization model and its corresponding dual problem are established which clearly show the pricing mechanism. Three scenarios have been studied. Bilateral contract between WH generators and user for different amount of power at different hour is introduced in the model in all the cases. Wind and hydro generators are committed to meet a firm commitment to an hourly dispatch schedule, in a network environment. Sellers offer into the market and they settle on the market-final price outcome and their previous contracted amount through contract for difference (cfd.). A notional generator of small capacity (thermal generator here) may be needed at some intervals to meet the WH schedule, as we used a thermal generator at bus 7.

In case 1 there is no loss and congestion. The wind generators are dispatchable and cleared against their offer prices. The results clearly show relationship between the LMP and the marginal price of the WH generators and show that the constraint-on payment which is the difference between the marginal generation price and the LMP at the bus is necessary. This is for forcing the renewable generations, instead of cheaper generation, to cater the contractual demands at some hours.

The results of case 2 show the effect on LMPs due to binding transmission constraint. The LMP is reduced (because of less expensive generations) at the "flow from" region and LMP at the "flow to" region is increased (because of more expensive generations).

Wind generators are treated as "Must Run" in case 3. The results of case 3 show that the dispatch cost is the minimum of these 3 cases. The LMPs are same as in case 1. The dispatch method in case 3 is currently used by New Zealand electricity market. At present, there is no problem because the wind penetration is not high enough (700MW or so). The intermittency of wind generation is absorbed by other generators. But in the future, the wind generation needs to be "firm" with the help of some energy storage devices like hydro generation or battery.

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DISCLAIMER

The views expressed in this paper do not necessarily reflect the views of Wintec.

REFERENCES

[1] E. D. Castronuovo and J. A. Pecas Lopes ," On the optimization of the Daily Operation of a Wind-Hydro Power Plant", IEEE Transactions on Power Systems, VOL. 19, No 3, August 2004. pp. 1599-1606.

[2] Lu, Y. V. Makarov, Y. Zhu, Ning Lu, N. P. Kumar, B. B. Chakrabarti "Unit Commitment Considering Generation Flexibility and Environment Constraints", proc. IEEE GM Conference, Minneapolis, 2010.

[3] G. N Bathurst, G. Strbac, "Value of combining energy storage and wind in short-term energy and balancing markets", Electrical Power Systems Research 67(2003) pp. 1-8. available://ww.sciencedirect.com.

[4] G. Tina, C. Brunetto, G. Foti, "Wind Generation and Hydrogen Storage Management in liberalized Electricity Market", CIRED, proc. 19th International Conference on Electricity Distribution", Paper 0261, Vienna, 21-24 May 2007

[5] Bhujanga B. Chakrabarti, N. Newham, D. Goodwin and C. Edwards, "Wind-Hydro Firming with Environmental Constraints in New Zealand, proc. IEEE GM Conference, Detroit, 2011.

[6] Bhujanga B. Chakrabarti, and Doug Goodwin, "Dispatch of Firm Wind Generation with Transmission Constraint", proc. IEEE GM Conference, San Diago, 2013.

[7] Thamer Althobaiti " Wind generation integration in New Zealand" Beng Tech Reaserch Project , CEID, Wintec, 2018

[8] GAMS Users' guide, GAMS Development Corporation, Washington, DC, USA, 2007. Available: www.gams.com.

BIOGRAPHIES

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